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C. H. Chang

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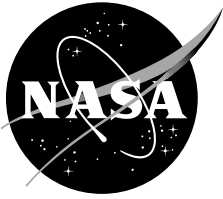
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C. H. Chang
Thermosciences Institute, Ames Research Center, Moffett Field, California

National Aeronautics and
Space Administration

Ames Research Center
Moffett Field, California 94035-1000

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C. H. Chang

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Ames Research Center

SUMMARY

The relationship between Joule heating, diffusion fluxes, and friction forces has been studied for both total and electron thermal energy equations, using general expressions for multicomponent diffusion in two-temperature plasmas with the velocity dependent Lorentz force acting on charged species in a magnetic field. It is shown that the derivation of Joule heating terms requires both diffusion fluxes and friction between species which represents the resistance experienced by the species moving at different relative velocities. It is also shown that the familiar Joule heating term in the electron thermal energy equation includes artificial effects produced by switching the convective velocity from the species velocity to the mass-weighted velocity, and thus should not be ignored even when there is no net energy dissipation.

1. INTRODUCTION

Electrical current in a plasma that is a mixture of different types of ions, electrons, and neutral species produces energy dissipation, i.e., Joule heating, which directly contributes to the thermal energy or temperature of plasmas. This term is usually represented as $\mathbf{J}_q \cdot \mathbf{E}$ in energy equations, where $\mathbf{J}_q$ and $\mathbf{E}$ represent electrical current density and electric field, respectively. In the presence of a magnetic field $\mathbf{B}$, the Joule heating term includes the effect of the Lorentz force with results $\mathbf{J}_q \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$, where $\mathbf{u}$ is the mass-averaged velocity of the plasma.

Due to the small mass of electrons, collisional energy transfer is inefficient between free electrons and heavy particles. As a result, persistent temperature differences often exist between free electrons and heavy particles, and this thermal nonequilibrium requires a separate energy equation for free electrons to be included in the analysis. Joule heating in the electron energy equation presented in textbooks (refs. 1, 2) takes the form of $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$, where q_i, $\mathbf{J}_i = \rho_i (\mathbf{u}_i - \mathbf{u})$, and $\mathbf{u}_i$ are respectively the electrical charge per unit mass, diffusive mass flux, and average velocity of species i, and subscript e denotes free electrons. Since $q_e \mathbf{J}_e$ is the electrical charge carried by diffusion flux, $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ appears to represent the work done by the motion of electrical charge through $\mathbf{E}$ and $\mathbf{B}$.

A question arises, however, regarding the origin of these terms. Forces acting on the charged species due to $\mathbf{E}$ and $\mathbf{B}$ are body forces, and terms representing the work done by body forces should *not* appear in thermal energy equations. This is precisely the reason that the work done by gravitational forces does not appear in thermal energy equations. It simply drops out when the mechanical energy equation for potential and kinetic energies, i.e., Bernoulli's equation, is subtracted from the total energy equation to obtain the thermal energy equation. Joule heating, however, undeniably contributes to the thermal energy, and its appearance in the thermal energy equation is therefore physically correct, contradicting the fact that the work done by body forces such as electromagnetic forces should *not* appear.

In the electron thermal energy equation widely used in high-temperature gas dynamics (ref. 3), the Joule heating term is simply treated as work done by body force (electric field) and added as a source term in the electron thermal energy equation. Also in a popular magnetohydrodynamic (MHD) description (ref. 1), the $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term is brought into the electron total energy equation as the work done by body forces, and kept in the thermal energy equation by simply ignoring the electron inertia rather than subtracting the mechanical energy from the total energy. Obviously, both of these approaches are unsatisfactory for the reasons explained above.

In contrast, multifluid descriptions (refs. 4, 5) treat the energy dissipation caused by the velocity difference between electrons and ions as the sole source of Joule heating instead of the work done by body forces. The physical mechanism of Joule heating is correctly represented in those descriptions, since Joule heating is not related to the work done by body forces, but to the frictional dissipation, as shown in this paper. In order to represent Joule heating as $\mathbf{J}_q$, $\mathbf{E}$ and $\mathbf{B}$, however, it is necessary to include additional terms involving diffusion fluxes brought into the system by switching convective velocity of the species from $\mathbf{u}_i$ to $\mathbf{u}$. It is not, in general, possible to simply replace the frictional dissipation term to $\mathbf{J}_q \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$.

Another question arises regarding the $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in the electron thermal energy equation, in particular, for the special case of zero electrical current density, i.e., $\mathbf{J}_q = \Sigma_i \, q_i \, \mathbf{J}_i = \mathbf{0}$, which is commonly referred to as "ambipolar diffusion." During the recent development of formulations for multicomponent diffusion in plasmas (refs. 6–8), it has been shown that concentration gradients of the charged species in plasmas result in nonzero $\mathbf{E}$, even when $\mathbf{J}_q = \mathbf{0}$. Here, the Joule heating term in the total thermal energy equation is, of course, zero, since $\mathbf{J}_q = \mathbf{0}$. The $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in the electron thermal energy equation, however, appears to be nonzero even in this special case. This counterintuitive behavior can also be explained by examining the relationship between diffusion and energy dissipation due to friction between species.

This paper presents a study of the relationships between Joule heating, diffusion fluxes, and friction between species for total and electron thermal energy equations. It is shown that Joule heating results from the combination of both diffusion fluxes and friction between species, which represents the resistance experienced by the species moving at different relative velocities. Use of the hydrodynamic theory of diffusion (refs. 8–10), instead of the standard kinetic theory (refs. 11–16), has provided a clear and simple physical interpretation of complex phenomena of multicomponent diffusion and frictional interaction between species in two-temperature plasmas with nonzero electrical current and electromagnetic fields. Section 2 presents a derivation of the total and electron thermal energy equations from the species thermal energy equation. The Joule heating terms in total and electron thermal energy

equations are constructed from friction forces *and* diffusion fluxes brought into the system by artificially switching convective velocities from $\mathbf{u}_i$ to $\mathbf{u}$. In addition to the real energy dissipation (Joule heating), the $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in the electron thermal energy equation therefore includes this artificial component, and should *not* be ignored even when $\mathbf{J}_q = \mathbf{0}$. Section 3 presents physical interpretations regarding the relationships between Joule heating, diffusion fluxes, and friction between species. Section 4 contains a few concluding remarks.

2. JOULE HEATING IN ENERGY EQUATIONS

Chemical reactions and viscous dissipation are not included for simplicity in the present analysis, since they are not directly related to Joule heating and diffusion fluxes. The thermal energy equation for each species i is then given by (ref. 13)

$$\frac{\partial(\rho_i e_i)}{\partial t} + \nabla \cdot (\rho_i e_i \mathbf{u}_i) = -\nabla \cdot \mathbf{q}_i - p_i \nabla \cdot \mathbf{u}_i + \sum_j Q_{ij} + \sum_j \zeta_{ij}(\mathbf{u}_j - \mathbf{u}_i) \cdot \mathbf{F}_{ij} \tag{1}$$

where ρ_i and p_i are respectively the partial mass density and partial pressure of species i; $e_i(T_i)$ is the partial specific internal energy of species i at temperature T_i; and $\mathbf{q}_i$ represents the conduction heat flux vector. $Q_{ij} = -Q_{ji}$ is the energy exchange between species i and j, and $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ is the mean "friction" force per unit volume of species j on species i with the j summation extending over N components in the mixture. Note that $Q_{ii} = 0$ and $\mathbf{F}_{ii} = \mathbf{0}$. General expressions for heat flux vector $\mathbf{q}_i$ have apparently not been derived in full generality, but results are available in particular cases (refs. 2, 4, 14–16). Nevertheless, it is not necessary to know explicit forms of $\mathbf{q}_i$ and Q_{ij} for current purposes. $(\mathbf{u}_j - \mathbf{u}_i)\cdot\mathbf{F}_{ij}$ represents the energy dissipation by the friction between species i and j, and ζ_{ij} is its fraction into species i which is given by (ref. 13)

$$\zeta_{ij} = \frac{m_j}{m_i + m_j} \tag{2}$$

where m_i is the mass of a single particle of species i. $\mathbf{F}_{ij}$ is of the general form (ref. 8)

$$\mathbf{F}_{ij} = \alpha_{ij} \cdot (\mathbf{u}_j - \mathbf{u}_i) + \gamma_{ij} \tag{3}$$

$$\gamma_{ij} = \beta_{ij} \cdot \nabla \ln T_j - \beta_{ji} \cdot \nabla \ln T_i \tag{4}$$

where α_{ij} and β_{ij} are frictional and thermophoretic force coefficients respectively, which now become tensors due to the presence of $\mathbf{B}$ (ref. 4). α_{ij} is related to binary diffusion coefficient tensors $\mathbf{D}_{ij}$ as (ref. 8)

$$\mathbf{D}_{ij} = p z_i z_j \alpha_{ij}^{-1} \tag{5}$$

where $z_i = p_i/p$. Derivation of equation (1) involves all body forces, including electromagnetic forces (ref. 13).

Although energy dissipation due to friction forces between species is represented by $(\mathbf{u}_j - \mathbf{u}_i)\cdot\mathbf{F}_{ij}$, the Joule heating term represented by $\mathbf{J}_q$, $\mathbf{E}$, and $\mathbf{B}$ does not explicitly show up in equation (1) as expected, for Joule heating is *not* related to the work done by the body force. Joule heating represented as $\mathbf{J}_q\cdot(\mathbf{E} + \mathbf{u}\times\mathbf{B})$ or $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u}\times\mathbf{B})$ appears only in single-fluid or MHD descriptions as a result of switching the convective velocity from $\mathbf{u}_i$ to $\mathbf{u}$, as the following development shows. First, equation (1) is rewritten with $\mathbf{u}$ as the convective velocity, with results

$$\frac{\partial(\rho_i e_i)}{\partial t} + \nabla\cdot(\rho_i e_i \mathbf{u}) = -\nabla\cdot(\mathbf{q}_i + h_i\mathbf{J}_i) - p_i\nabla\cdot\mathbf{u} + \frac{\mathbf{J}_i}{\rho_i}\cdot\nabla p_i + \sum_j Q_{ij} + \sum_j \zeta_{ij}(\mathbf{u}_j - \mathbf{u}_i)\cdot\mathbf{F}_{ij} \qquad (6)$$

where $h_i = e_i + p_i/\rho_i$ is the partial specific enthalpy of species i.

A. Total Thermal Energy Equation

The total thermal energy equation is obtained by summing equation (6) over all species, with the result

$$\frac{\partial(\rho e)}{\partial t} + \nabla\cdot(\rho e \mathbf{u}) = -\nabla\cdot\sum_i(\mathbf{q}_i + h_i\mathbf{J}_i) - p\nabla\cdot\mathbf{u} + \sum_i \frac{\mathbf{J}_i}{\rho_i}\cdot\nabla p_i + \sum_{ij} \zeta_{ij}(\mathbf{u}_j - \mathbf{u}_i)\cdot\mathbf{F}_{ij} \qquad (7)$$

where $e = (1/\rho)\Sigma_i\rho_i e_i$ is the total thermal energy, $p = \Sigma_i p_i$ is the total pressure, and use has been made of $\Sigma_{ij}Q_{ij} = 0$. Derivation of the Joule heating term consisting of $\mathbf{J}_q$, $\mathbf{E}$, and $\mathbf{B}$ requires both $\Sigma_i(1/\rho_i)\,\mathbf{J}_i\cdot\nabla p_i$ and $\Sigma_{ij}\,\zeta_{ij}\,(\mathbf{u}_j - \mathbf{u}_i)\cdot\mathbf{F}_{ij}$ in equation (7), as shown below.

In the limit of large friction between species, in which diffusional behavior results, the partial pressure gradient is given by (ref. 8)

$$\frac{\nabla p_i}{\rho_i} = -\frac{D\mathbf{u}}{Dt} + \mathbf{F}_i + \frac{1}{\rho_i}\sum_j \mathbf{F}_{ij} \qquad (8)$$

where $D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla$, and $\mathbf{F}_i$ is the body force per unit mass acting on species i which is taken to be of the form

$$\mathbf{F}_i = \mathbf{g} + q_i(\mathbf{E} + \mathbf{u}_i\times\mathbf{B}) \qquad (9)$$

where $\mathbf{g}$ is the acceleration of gravity.

Using equation (8), it is shown that

$$\sum_i \frac{\mathbf{J}_i}{\rho_i}\cdot\nabla p_i = \sum_i q_i\mathbf{J}_i\cdot(\mathbf{E} + \mathbf{u}_i\times\mathbf{B}) + \sum_{ij}(\mathbf{u}_i - \mathbf{u})\cdot\mathbf{F}_{ij} \qquad (10)$$

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Use has been made of $\Sigma_i \mathbf{J}_i = \mathbf{0}$. It is also shown that

$$\mathbf{J}_i \cdot (\mathbf{u}_i \times \mathbf{B}) = \mathbf{J}_i \cdot \left(\frac{\mathbf{J}_i}{\rho_i} \times \mathbf{B} + \mathbf{u} \times \mathbf{B} \right) = \mathbf{J}_i \cdot (\mathbf{u} \times \mathbf{B}) \tag{11}$$

Frictional terms are combined together with the result

$$\sum_{ij} \zeta_{ij} (\mathbf{u}_j - \mathbf{u}_i) \cdot \mathbf{F}_{ij} + \sum_{ij} (\mathbf{u}_i - \mathbf{u}) \cdot \mathbf{F}_{ij} = \sum_{ij} \frac{m_i \mathbf{u}_i + m_j \mathbf{u}_j}{m_i + m_j} \cdot \mathbf{F}_{ij} \equiv \sum_{ij} A_{ij} \tag{12}$$

Observe that $\Sigma_{ij} A_{ij} = 0$, since $A_{ij} = - A_{ji}$. From equations (10)–(12), it is shown that

$$\sum_i \frac{\mathbf{J}_i}{\rho_i} \cdot \nabla p_i + \sum_{ij} \zeta_{ij} (\mathbf{u}_j - \mathbf{u}_i) \cdot \mathbf{F}_{ij} = \mathbf{J}_q \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{13}$$

The total thermal energy equation then becomes

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho e \mathbf{u}) = -\nabla \cdot \sum_i (\mathbf{q}_i + h_i \mathbf{J}_i) - p \nabla \cdot \mathbf{u} + \mathbf{J}_q \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{14}$$

Equation (14) is the standard total thermal energy equation which can be found in textbooks (refs. 1, 2). As shown above, construction of Joule heating requires both (1) frictional forces that represent the collisional coupling between dissimilar species moving at different relative velocities and (2) diffusion fluxes brought into the equation by switching the convective velocity from species velocity $\mathbf{u}_i$ to the mass-averaged velocity $\mathbf{u}$.

B. Electron Thermal Energy Equation

The electron thermal energy equation is obtained by simply replacing the subscript i with e in equation (6) with the result

$$\frac{\partial (\rho_e e_e)}{\partial t} + \nabla \cdot (\rho_e e_e \mathbf{u}) = -\nabla \cdot (\mathbf{q}_e + h_e \mathbf{J}_e) - p_e \nabla \cdot \mathbf{u} + \frac{\mathbf{J}_e}{\rho_e} \cdot \nabla p_e + \sum_i Q_{ei} + \sum_i \zeta_{ei} (\mathbf{u}_i - \mathbf{u}_e) \cdot \mathbf{F}_{ei} \tag{15}$$

Note that equation (15) contains $(1/\rho_e) \mathbf{J}_e \cdot \nabla p_e$ and $\Sigma_i \zeta_{ei} (\mathbf{u}_i - \mathbf{u}_e) \cdot \mathbf{F}_{ei}$ instead of the more familiar $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term as in equation (5.2) of section IV in reference 2. As in the case of the total thermal energy equation, the $q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term is obtained by combining $(1/\rho_e) \mathbf{J}_e \cdot \nabla p_e$ and $\Sigma_i \zeta_{ei} (\mathbf{u}_i - \mathbf{u}_e) \cdot \mathbf{F}_{ei}$ and by further simplifications as shown below.

Replacing ∇p_e in equation (15) using equation (8) would not make equation (15) simpler, since more terms would be brought in. Nevertheless, it is possible to seek further simplification by exploiting the fact that the electron mass m_e is very small compared to the masses of heavy particles. This can be done

by neglecting terms of order $\varepsilon^2 \equiv m_e$ in comparison to terms of order unity. Equation (8) for electrons then becomes

$$\frac{\nabla p_e}{\rho_e} = q_e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + \frac{1}{\rho_e}\sum_i \mathbf{F}_{ei} \tag{16}$$

Using equation (11), it is shown that

$$\frac{\mathbf{J}_e}{\rho_e} \cdot \nabla p_e = q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \frac{\mathbf{J}_e}{\rho_e} \cdot \sum_i \mathbf{F}_{ei} \tag{17}$$

$\mathbf{F}_{ei}$ is not ignored, since α_{ei} is of order $\varepsilon \equiv \sqrt{m_e}$ (refs. 7, 8). Using equation (17) and $\zeta \approx 1$, equation (15) then becomes

$$\frac{\partial(\rho_e e_e)}{\partial t} + \nabla \cdot (\rho_e e_e \mathbf{u}) = -\nabla \cdot (\mathbf{q}_e + h_e \mathbf{J}_e) - p_e \nabla \cdot \mathbf{u} + q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

$$+ \sum_i Q_{ei} + \sum_i (1/\rho_i)\mathbf{J}_i \cdot \mathbf{F}_{ei} \tag{18}$$

Equation (18) becomes essentially the same as equation (5.2) of section IV in reference 2, when $\Sigma_i (1/\rho_i) \mathbf{J}_i \cdot \mathbf{F}_{ei}$ is ignored, which can be done by neglecting either $\mathbf{J}_i$ or $\mathbf{F}_{ei}$, although there is no apparent justification for doing so.

As briefly described in section 1, the $q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term, originally brought into the system as the work done by body forces, is kept in the electron thermal energy equation with $\mathbf{u}_e$ as the convective velocity (eq. (5.196) in ref. 1). This is done by ignoring the inertia of electrons instead of following the more general practice of subtracting the mechanical energy from the total energy, which would, of course, eliminate the $q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in the thermal energy equation. However, equation (17) illustrates that ignoring electron inertia also means that $q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ drops out anyway from equation (5.196) of reference 1 by canceling with the $(\mathbf{u}_e - \mathbf{u}) \cdot \nabla p_e$ term, which is also present in equation (5.196) of reference 1. (All frictional interactions ($\mathbf{F}_{ei}$) are also neglected when electron inertia is neglected in reference 1, which would result in an ideal MHD description without net energy dissipation as shown below.) By using $\mathbf{u}$ as the convective velocity, the $q_e\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term is, of course, brought back into the electron thermal energy equation, as the preceding development shows.

Further simplification could be explored by neglecting terms of order $\varepsilon \equiv \sqrt{m_e}$ as well. Equation (8) then becomes

$$\frac{\nabla p_e}{\rho_e q_e} = \mathbf{E} + \mathbf{u}_e \times \mathbf{B} + \frac{1}{\rho_e q_e}\sum_i \gamma_{ei} \tag{19}$$

Observe that $\gamma_{ei} = \beta_{ie} \cdot \nabla \ln T_e$, since β_{ie} is order unity, and β_{ei} is order ε^2 (ref. 17). Using equation (19), equation (15) is rewritten, with results

$$\frac{\partial(\rho_e e_e)}{\partial t} + \nabla \cdot (\rho_e e_e \mathbf{u}) = -\nabla \cdot (\mathbf{q}_e + h_e \mathbf{J}_e) - p_e \nabla \cdot \mathbf{u} + q_e \mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

$$+ \sum_i Q_{ei} + \sum_i (1/\rho_i)\mathbf{J}_i \cdot \gamma_{ei}$$

(20)

Equation (20) is not much simpler than equation (18), unless thermal diffusion of electrons is negligible. Furthermore, this simplification of ignoring terms of order ε, would result in an ideal MHD description in which resistive effects vanish, since the electrical conductivity of the plasma σ is itself of order ε^{-1} (refs. 8, 12). Observe that equation (19) is essentially equivalent to the $\sigma \to \infty$ limit of equation (9.47) of reference 18, which indeed confirms the passage into the realm of ideal MHD. (In the present context, however, the term "ideal" does not imply that the plasma flow is isentropic. Diffusion and finite-rate chemical reactions are still irreversible processes.) Note that it still is possible to keep resistive effects by ignoring only terms of order ε^2, while keeping terms of order ε, which is precisely the way σ is represented by the binary diffusion coefficients of electrons and ions (ref. 12).

3. JOULE HEATING IN MHD

It can easily be explained that the acceleration of the charged species due to electromagnetic forces and the resulting increase of the kinetic energy would eventually convert to the increase in random thermal velocities through collisions (dissipation) between dissimilar species. Friction between the same species is zero ($\mathbf{F}_{ii} = \mathbf{0}$), and applying body forces such as gravitational or electromagnetic field does *not* result in nonzero $\mathbf{F}_{ii}$, since all the particles of the same species are subject to the same amount of forces and acceleration. Therefore, applying $\mathbf{g}$, $\mathbf{E}$, or $\mathbf{B}$ merely changes $\mathbf{u}_i$ rather than random thermal velocities, and does *not* contribute to the thermal energy without a proper dissipation mechanism such as friction between dissimilar species. In plasmas composed of both positively and negatively charged particles, particles with positive charges are accelerated by $\mathbf{E}$ and $\mathbf{B}$ in the *opposite* direction of the acceleration of particles with negative charges, increasing velocity differences which in turn produces electrical current. This increase in velocity differences is balanced by increased frictional forces represented as resistivity. In other words, the "extra" source of acceleration or driving forces due to $\mathbf{E}$ and $\mathbf{B}$ is balanced with "extra" frictional forces, and continued supply of force and energy by $\mathbf{E}$ and $\mathbf{B}$ results in the increase of thermal energy by friction. This clearly shows that the resistance experienced by species moving at different velocities is an essential mechanism in Joule heating.

As shown in the preceding sections, however, diffusion fluxes are also necessary in order to represent the Joule heating in terms of $\mathbf{J}_q$, $\mathbf{E}$, and $\mathbf{B}$ in the energy equations. These diffusion fluxes are brought into the system of single-fluid or MHD descriptions, since species in the plasma in these descriptions are thought to be moving with the convective velocity of the mass-weighted velocity $\mathbf{u}$ of the mixture rather than with their own velocities $\mathbf{u}_i$. If $\mathbf{u}_i$ is kept as the convective velocity, there is no Joule heating term consisting of $\mathbf{J}_q$, $\mathbf{E}$, and $\mathbf{B}$ in the energy equations as in equation (1), consistent with the fact that the work produced by body forces should *not* appear in thermal energy equations.

In multifluid descriptions, in which each species has its own momentum and energy equations, Joule heating terms do not explicitly show up in the species thermal energy equations. Nevertheless, Joule

heating is provided to the system by acceleration terms in the species momentum equation and frictional dissipation in the species energy equation, as explained above. The electron energy equation in widely used multifluid descriptions (refs. 4, 5) involves the frictional dissipation in the system, consistent with the mechanism described above. However, simple conversion of the frictional term to the Joule heating as in equation (2.18) of reference 4 is valid only when ∇p_e is ignored, as shown in equation (17). This can also be easily observed by inspecting equations (2.2e) and (2.18) in reference 4.

In contrast, the increase in differences between species velocities due to the electromagnetic field is not represented by the single momentum equation for the whole mixture used in single-fluid or MHD descriptions (refs. 1, 2). Diffusion fluxes nevertheless include the increase in species velocity differences caused by extra driving forces due to $\mathbf{E}$ and $\mathbf{B}$ which are, of course, balanced with friction between species as described above. The species velocity differences and frictional dissipation eventually become the Joule heating term in energy equations, when diffusion fluxes are combined with the frictional dissipation terms as shown in the preceding section.

When $\mathbf{J}_q = \mathbf{0}$, the Joule heating term in the total thermal energy equation is, of course, zero. However, the $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in equation (18) or (20) does not vanish even in this special case as the question raised in section 1 shows. The fact that concentration gradients of the charged species in plasmas result in nonzero $\mathbf{E}$, even when $\mathbf{J}_q = \mathbf{0}$, can easily be illustrated by equation (19). In particular, in the absence of magnetic field and temperature gradient, equation (19) simply becomes

$$\mathbf{E} = \frac{\nabla p_e}{\rho_e q_e} \tag{21}$$

This has the simple and intuitive interpretation that electrons are in an electrostatic equilibrium with the electric field $\mathbf{E}$, which is precisely analogous to hydrostatic equilibrium in a gravitational field. $\mathbf{E}$ in equation (21) is not induced by charge separation as discussed in a previous study (ref. 19). On the contrary, charge neutrality in the plasma is preserved by this $\mathbf{E}$ (refs. 6, 7).

The preceding development shows that $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ is obtained by combining the frictional dissipation represented by $\Sigma_i \zeta_{ei} (\mathbf{u}_i - \mathbf{u}_e)\cdot\mathbf{F}_{ei}$ with $(1/\rho_e) \mathbf{J}_e\cdot\nabla p_e$ which is artificially brought into the system by switching the convective velocity from $\mathbf{u}_e$ to $\mathbf{u}$. Since $(1/\rho_e) \mathbf{J}_e\cdot\nabla p_e$ is in general nonzero, the $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term obviously is nonzero as well. In other words, the $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term involves an artificial effect of switching the convective velocity in addition to the energy dissipation by friction, and thus should be included in the electron thermal energy equation even when $\mathbf{J}_q = \mathbf{0}$.

Note that equations (19) and (21) are derived by neglecting the terms of order ε in equation (16). Since the frictional force coefficient α_{ei} is also of order ε, α_{ei} vanishes from the system, which in turn results in infinite electron diffusion coefficients (eq. (5)) and electrical conductivity (refs. 8, 12). In other words, zero m_e simply deactivates frictional interaction between electrons and other species, and frictional dissipation and resistive effects vanish. In this simplification, $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ merely replaces $(1/\rho_e) \mathbf{J}_e\cdot\nabla p_e$ and no energy is dissipated by this term.

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When $\mathbf{J}_q$ and m_e are not zero, the correct amount of Joule heating is produced by the "extra" diffusion of charged species induced by "extra" driving forces, as explained above. This can be automatically accomplished, when $\mathbf{J}_i$, $\mathbf{E}$, and $\mathbf{B}$ are consistently determined by solving the generalized Stefan-Maxwell equations (ref. 8) involving nonzero m_e and Maxwell's relations in a coupled manner. The solution of these equations determines $\mathbf{E}$ as well as $\mathbf{J}_i$, and this implicitly determines the relation between $\mathbf{E}$ and $\mathbf{J}_q$, i.e., Ohm's law. This shows that Ohm's law is not an independent constitutive relation in multicomponent MHD, and it is not, in general, possible to express $\mathbf{J}_q$ directly in terms of $\mathbf{E}$, e.g., "generalized" Ohm's law, without determining $\mathbf{J}_i$.

4. CONCLUDING REMARKS

A study has been presented of the relationship between diffusion fluxes, friction forces, and Joule heating in two-temperature multicomponent MHD, using hydrodynamic theory of diffusion (refs. 8–10) which provides a clear physical interpretation. It has been shown that energy produced by $\mathbf{J}_q$ flowing through $\mathbf{E}$ and $\mathbf{B}$ contributes to thermal energy via additional driving forces for diffusion by $\mathbf{E}$ and $\mathbf{B}$ and the resistance experienced between dissimilar species moving at different relative velocities rather than the work done by body forces as described in previous studies (refs. 1–3). It is a coincidence that the $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term brought into the electron thermal energy equation as the work done by body forces (refs. 1–3) is identical to the term derived in this paper from frictional dissipation and diffusion fluxes.

Derivations of both total and electron thermal energy equations have also been presented, and it has been illustrated that the derivation of Joule heating terms requires both diffusion fluxes and friction forces between species. The $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ term in the electron thermal energy equation involves the $(1/\rho_e)\ \mathbf{J}_e\cdot\nabla p_e$ term, which is artificially brought into the system by switching the convective velocity from $\mathbf{u}_e$ to $\mathbf{u}$. $(1/\rho_e)\ \mathbf{J}_e\cdot\nabla p_e$ is not, in general, negligible, and $q_e\mathbf{J}_e\cdot(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ therefore should not be ignored, even when $\mathbf{J}_q = \mathbf{0}$.

Ignoring frictional interaction between species, in particular between electrons and other species, leads to equations (19) and (21). Evaluating $\mathbf{E}$ using equation (19) or (21) results in an ideal MHD description, and thus should be avoided to keep resistive effects. The proper amount of Joule heating would automatically be produced when $\mathbf{J}_i$, $\mathbf{E}$, and $\mathbf{B}$ are evaluated by solving generalized Stefan-Maxwell equations (ref. 8) involving nonzero m_e and Maxwell's relations.

It is not obvious that the $\Sigma_i\ (1/\rho_i)\ \mathbf{J}_i\cdot\mathbf{F}_{ei}$ term in equation (18) could indeed be neglected, especially when $\mathbf{J}_q \approx \mathbf{0}$, where electron and heavy particle diffusion velocities would be comparable. (The $\Sigma_i\ \zeta_{ei}\ (\mathbf{u}_i - \mathbf{u}_e)\cdot\mathbf{F}_{ei}$ term in equation (15) is not negligible, since diffusion velocity of electrons for nonzero $\mathbf{J}_q$ can be large.) Nevertheless, evaluation of those terms does not represent additional complexity in multicomponent MHD, since friction coefficients α_{ij} (or diffusion coefficients $\mathbf{D}_{ij}$) and diffusion fluxes $\mathbf{J}_i$ are obtained anyway, when species conservation equations are solved.

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C. H. Chang

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The relationship between Joule heating, diffusion fluxes, and friction forces has been studied for both total and electron thermal energy equations, using general expressions for multicomponent diffusion in two-temperature plasmas with the velocity dependent Lorentz force acting on charged species in a magnetic field. It is shown that the derivation of Joule heating terms requires both diffusion fluxes and friction between species which represents the resistance experienced by the species moving at different relative velocities. It is also shown that the familiar Joule heating term in the electron thermal energy equation includes artificial effects produced by switching the convective velocity from the species velocity to the mass-weighted velocity, and thus should not be ignored even when there is no net energy dissipation.

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